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Hands-on Math is a resource book of math activities involving manipulatives and is designed to supplement any $\mathrm{K}-1$ math program. It is built on the premise that young children learn math concepts best through hands-on experiences with concrete objects. As you implement the activities, you will see how using manipulatives actively involves children in the learning process and gives them a wonderful sense of discovery and empowerment as they explore mathematical concepts.

The core of the book consists of activities for children to reinforce math skills. The activities are divided into five major sections based on NCTM national math standards: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. Each section contains two opening pages, which list the skills taught in that section, bulletin board ideas, and ideas for setting up one or more learning centers. At the end of the book, you'll find a list of related literature. These books can be kept at learning centers to read aloud, to introduce or reinforce math concepts, or to integrate math with literature.

It is important to note that the activities are designed to lead children step-bystep from working with concrete objects to constructing pictorial representations of objects to using symbols such as numerals. For example, in students' first experiences with addition, they should combine sets of real objects, such as three crayons and two crayons. The next step is to record their experiences with pictures.


Finally, they describe the operation using symbols ( $3+2=5$ ). In all cases, children will need guidance from the teacher to connect the manipulative activities and games with abstract symbols.

Use this resource to cover standards with a hands-on approach, and watch children learn math skills while having fun!

## Getting Started <br> Management Tips and Using Manipulatives

An activity-based program requires more materials and promotes more movement and verbal interaction than a program using work sheets or math texts. There are various ways to integrate manipulative activities into your math program. Some activities are appropriate for the whole class, while others may be better suited to small groups. Or you may introduce some activities to the whole group before placing the activities at a learning center for independent use.

Workstations are another alternative. Stations can be set up around
 the classroom, with one activity at each station. For example, when studying shapes, set up five stations where children can do shape rubbings, use pattern block shapes, make cut-and-paste shape pictures, form shapes on geoboards, and play a shape-matching game. Children could rotate to all of the stations in one day, or they could visit a different station each day.

For teachers who are ready to make manipulatives the focus of their math program, here are some management tips that will help:

- Allow children plenty of time to informally "play" with the manipulatives before using them in directed lessons so that later on they will be able to focus on mathematical concepts rather than on the manipulatives themselves. The amount of time set aside for exploration will depend on the needs of your students. You will find, however, that as children develop intellectually and reach new levels of understanding, they will benefit from repeated free-exploration opportunities throughout the year.
- Introduce math manipulatives gradually. Discuss and model the safe and responsible use of these materials. It is also a good idea to rotate the materials and activities offered at any one time.
- Find a place to store the manipulatives so that they are easily accessible to the children. Label shelves, drawers, and cabinets with words, codes, or pictures so that children can take responsibility for cleaning up.
- Establish ground rules for math activity time, display the rules prominently, and have children role-play the rules. Enforce these rules consistently, and praise responsible behavior.
- Model each activity with the whole class or with a small group before asking children to do the activity independently.



## Getting <br> Started <br> Collecting Materials

The materials in Hands-on Math are a combination of inexpensive, everyday objects and commonly available commercial math manipulatives, such as pattern blocks and Unifix cubes. As you read over the following lists of recommended materials, do not feel that you need to collect each item. Feel free to substitute materials available to you for those listed.

Start by enlisting the help of children and parents in collecting inexpensive materials. (There are many suggestions for the use of these "math collections" in the following chapters.) At the beginning of the year, send a letter home listing items you would like to collect. As the manipulatives arrive, place them all in a large container. Later, have children sort the materials into separate containers. Then label the containers, and place them in a well-marked, central location.


There are also many excellent commercial products available. Although they can be expensive, the cost of these materials can be defrayed in a number of ways:

- Ask your principal to allow funds earmarked for the purchase of workbooks to be used for the purchase of manipulatives instead.
- Write a proposal to the parent support group at your school asking for money or materials.
- Have a class fund-raising project, such as a garage sale or bake sale.
- As a learning center or a free-time activity, have the children in your class make some of the simpler manipulatives.
- Host an after-school or evening workshop for teachers, parents, and other volunteers. Ask participants to bring materials and spend a couple of hours making the manipulatives.
- In lieu of a gift exchange at holiday time, have each child present the classroom with a gift chosen from the list of needed materials.


## Getting <br> Started <br> Math Collections Materials List

## The following is a list of everyday materials that can be used for class math collections:

- Macaroni in various shapes
- Dry beans (e.g., pinto, navy, lima, black)
- Beads of all kinds
- Shells
- Buttons
- Nuts, seeds, rocks, stones, small pine cones, pods
- Bolts, nuts, washers
- Small lids, bottle caps
- Plastic bread tags
- Paper clips
- Keys
- Coins
- Plastic animals
- Small tiles
- Golf tees
- Spools
- Refrigerator magnets
- Small boxes for storing math collections
- Egg cartons for graphing and sorting
- Plastic food containers
- Food coloring to color macaroni
- Toothpicks
- Craft sticks
- Clothespins
- Old magazines and catalogs
- Containers for sorting (aluminum, Styrofoam, or microwave trays; pie plates; paper plates)
- Old hula hoops for concrete Venn diagrams
- Tens cups (nut, soufflé, medicine, or portion cups)
- Old measuring cups and spoons


Commonly available commercial products include:

- Pattern blocks
- Attribute blocks
- Cuisenaire rods
- Base-ten blocks
- Unifix cubes
- Tangrams
- Plastic counters
- Dice
- Spinners
- Geoboards
- Play money
- Demonstration clock
- Balance scale
- Kitchen scale
- Thermometer
- Hundreds board
- Timers
- 2 cm cubes
- Wooden stringing beads
- Floor graph
- Magnetic shapes
- Dominoes
- Rulers, yardsticks


## Number and <br> SKILLS LIST

 Sets and Quantities; Numbers and Symbols- Counts with understanding from 0 to 100
- Attains understanding of one-toone correspondence
- Recognizes "how many" in sets
- Understands that there are many ways to represent quantities: objects, word names, numbers
- Compares sets of objects as "greater than,""less than," or "equal to"
- Understands representation of standard numerals
- States relative position and magnitude of whole numbers between 0 and 100
- Understands the meaning of the symbols + , $=$, and -
- Understands and applies the concepts of counting (by twos, threes, fives, tens)
- Skip-counts using a number line
- Uses ordinal numbers
- Uses cardinal numbers

Counting • Provide a wide variety of objects for counting. When children are skilled at counting by ones, encourage them to group objects and count by twos, fives, or tens. The following objects may be used for counting:

- Manipulatives bagged in sets of 10 or 20
- Math manipulatives to be counted in large quantities
- Small paper cups
- Wooden beads (string and count)
- Pennies and a bank
- Plastic linking cubes
- Pattern blocks

Include a selection of counting books so children can read and count. Post a hundreds chart low enough so children can touch each number. Purchase or make a walk-on number line so children can count in a kinesthetic style. (Use heavy plastic colored tape and a permanent marker to make the number line.)


## BOLLHOMOBOABDODEAS

You Can Count on Us $\bullet$ Use this bulletin board to celebrate growth in continuing skills. Prepare the following cutouts for the whole class: a smile, a pair of eyes, a hand, and a pair of feet with toes. Divide the bulletin board space into four parts. Label the sections $15,25,55$, and 10 s. Each time a child demonstrates a new counting skill, write the child's name on the appropriate cutout, and place it on the board. Be sure to give children practice in these skills every day.


Hands-on Numerals $\leqslant$ Use the pictures created by children in the Feely Numerals activity (page 15) for a textured bulletin board display that is sure to have "hands-on" appeal. Have children sign their pictures and write or dictate a sentence telling how their number feels. Mount or frame each picture using brightly colored paper.



## Counting by Ones

Skills: counting, one-to-one correspondence
Give each child one set of objects (e.g., pennies, cubes, or beads). Ask children to count by ones as they drop pennies into a bank, snap together linking cubes, or string beads.


## Counting to Music

nez Skills: counting, one-to-one correspondence
Combine music, movement, and math skills when you play this game. Play lively music that has a steady beat. As the music plays, call out directions such as Tap your nose ten times, Knock your knees and count to 20, or March in place to 25 . Have children count aloud as they follow the directions and move to the music.


## Football Counting

nss Skill: counting
Count aloud by ones; then "pass" the counting by handing a child a football. Have the child continue counting where you left off. Then have that child pass the football to another child who will continue counting. Repeat the process until all children have had a chance to count.

## Count and Match

Stis Sills: counting, one-to-one correspondence
Divide the class into pairs, and have partners sit at the ends of one desk, facing each other. Give each child 10-15 objects from the math collections. Place a partition (e.g., a file folder or book) between them in the middle of the desk. Have one child display a set of objects on the desk. Then ask that child to lift the partition and let his or her partner look at the set for 15 seconds. Then have the partner display the same set on his or her side of the desk. Have children repeat the process, and have partners alternate turns.


## Same or Different?

nse Skill: comparison of quantities
Divide the class into pairs, and have partners sit at the ends of one desk, facing each other. Give each child 1-10 objects from the math collections. Place a partition (e.g., a file folder or book) between them in the middle of the desk. Tell both children to display a set of objects 1 through 10 on their side of the partition. Have one child lift the partition, and ask partners to compare their sets to see if they are the same or different quantities.

## Count, Count, Count

tex Skills: counting, one-to-one correspondence
Divide the class into groups of three or four. Give each group ten paper cups and objects (e.g., beans, buttons). Then ask children to count out ten objects in each cup. When they are finished, count with the class by tens to determine how many objects there were for each group. Have children repeat this activity counting by fives.


MATERIALS
$\checkmark$ math collections (see page 7)
$\checkmark$ file folder or book

MATERIALS
$\checkmark$ math collections (see page 7)
$\checkmark$ file folder or book

MATERIALS
$\checkmark$ small paper cups
$\checkmark$ multiples of inexpensive objects (beans, buttons, macaroni, toothpicks, seeds)

## MATERIALS

$\checkmark$ Counting Backward Animal Sign reproducible (page 19)
$\checkmark$ tongue depressor or wooden dowel

## MATERIALS

$\checkmark$ large number line
$\checkmark$ teacher pointing stick
$\checkmark$ individual number lines

## MATERIALS

$\checkmark$ large number line
$\checkmark$ teacher pointing stick
$\checkmark$ individual number lines

## Backward Counting

ns Skill: counting

Color, cut out, and laminate a copy of the Counting Backward Animal Sign reproducible. Attach the cutouts to a tongue depressor or 9 " ( 23 cm ) wooden dowel. Stand in front of the class, facing the children, and hold the "face" side of the animal sign for them to see. Have children count by ones in unison. Tell them that when you turn the sign so the back of the animal shows, they are to stop counting forward and begin counting backward. Explain that when you turn the sign around again so they can see the animal's "face," they are to count forward again. For example, if the class is counting $0,1,2,3,4$ and you turn the animal sign around, they should start counting backward 3, 2, 1, 0 Once the class has the idea, choose a child to direct the counting.

## Number-Line Counting

Skills: counting, one-to-one correspondence
Display a large number line, or write the numbers 0 to 20 on a large piece of butcher paper. Use the large number line to model how to count while walking to a specified number on the number line, beginning with 1. Then select a child to lead the class in repeating the process. Ask the child to select the starting number and lead the class in counting by ones to 20. Afterward, invite children to practice counting on their individual number lines.


## Counting Backward

Tes Skills: counting, one-to-one correspondence
Use the large number line to model how to count backward while pointing to a specified number on the number line. Then select a child to lead the class in repeating the process. Ask the child to select the starting number and lead the class in counting backward by ones from 20 initially and eventually from 100, as the class is able. Afterward, invite children to practice counting on their individual number lines.

